1. \( f(xy) = f(x)f(y) - f(x+y) + 1 \) and \( f(1) = 2 \).

Put \( x = 1 \) and \( y = n \): \( f(n) = 2f(n) - f(n+1) + 1 \Rightarrow f(n+1) = f(n) + 1 \). This means that \( f(n) = n + 1 \).

For \( x = k \) and \( y = \frac{n}{k} \): \( f(n) = f(k)f(\frac{n}{k}) - f(k + \frac{n}{k}) + 1 \Rightarrow n + 1 = (k + \frac{n}{k})f(\frac{n}{k}) - k - f(\frac{n}{k}) + 1 \Rightarrow f(\frac{n}{k}) = \frac{n+1}{k+1} \).

Thus, for \( x \in \mathbb{Q} \) \( f(x) = x + 1 \), and since \( \mathbb{Q} \) is dense in \( \mathbb{R} \) we have \( f(x) = x + 1 \) for \( x \in \mathbb{R} \).

2. Let \( f(0) \neq 0 \). Then \( |f(0) - 0| > |f(f(0)) - f(0)| \geq |f(f(f(0))) - f(f(0))| \geq |0 - f(0)| \geq |f(0) - f(f(f(0))))| = |f(0) - f(0)| \). Contradiction.

3. \( f(x) + f(\frac{x}{x-1}) = 1 + x(1) \).

For \( x = \frac{x-1}{x} \): \( f(\frac{x-1}{x}) + f(\frac{x}{x-1}) = 2\). For \( x = \frac{x-1}{x} \): \( f(\frac{x-1}{x}) + f(x) = \frac{x+1}{x} \).

\((1) + (3) - (2) \): \( 2f(x) = 1 + x + \frac{x}{x+2} - \frac{2x-2}{x} = \frac{x^2-x^2-1}{x^2(x-1)} \Rightarrow F(x) = \frac{x^3-x^2-1}{2x(x-1)} \).

4. \( 3333 = f(9999) = f(9996)+f(3)+\delta_{9996,3} = f(9993)+2f(3)+\delta_{9993,3} = \ldots = 3333f(3)+\delta_{9996,3}+\ldots \).

This means that \( f(3) = 1 \) and \( \delta_{9996,3} = \delta_{9993,3} = \ldots = 0 \) since \( 2013 \) is divisible by 3, \( f(2013) = 671 \).

Since the function is continuous it maps \((\infty, \infty)\) to some interval \( X \). All irrationals from \((-\infty, \infty)\) are mapped into rationals from \( X \). Thus, rationals from \((-\infty, \infty)\) are mapped to all irrationals and the set of rationals from \( X \). This is impossible, since the set of irrationals in \( X \) is countable, while the set of rationals is countable.

6. Let \( f \) be not equal to zero. Then, since \( f \) is continuous, there exists an interval \([a, b]\) such that \( a-b < 1 \), \( f(a) = 0 \) and \( f(b) \) is the maximum on the interval. By the Mean Value Theorem: \( \frac{f(b)-f(a)}{b-a} = f'(c) \Rightarrow |f'(c)| < |f(b)| < |f'(c)| \). Contradiction.

7. Let \( g(t) = \frac{f(\cos t)}{\sin t} \), then \( g(t+\pi) = g(t) \).

\( g(2t) = \frac{f(2\cos^2(t)-1)}{2\sin t \cos t} = \frac{f(\cos t)}{\sin t} = g(t) \).

Then for any \( n \) and \( k \) \( g(1+\frac{n\pi}{k}) = g(2k+n\pi) = g(2k) = g(1) \). Since, \( \{1+\frac{n\pi}{k} : n, k \in \mathbb{Z} \} \) is dense and \( g \) is continuous on its domain, \( g \) is constant on its domain. We know that \( g(t) = g(-t) \), thus \( g(t) = 0 \), when \( t \) is not a multiple of \( \pi \). Hence, \( f(x) = 0 \) for \( x \in (-1, 1) \). Finally, since \( f \) is continuous, \( f(x) = 0 \) for \( x \in [-1, 1] \).

8. When \( a > 2 \), \( f(x) = \frac{2a}{x^2} \) satisfies the perimeter and the area are equal to \( \frac{2a^3}{x^2} \).

Now, suppose \( a \leq 2 \). Let \( M \) be the maximal value of \( f(x) \). Then the area does not exceed \( a \cdot M \). At the same time, the perimeter is at least \( 2M + a \) from \((0, 0)\) to point with \( f(x) = M \), from point with \( f(x) = M \) to \((a, 0)\) and from \((a, 0)\) to \((0, 0)\). It can be seen that area \( \leq a \cdot M \leq 2M + a \leq \text{perimeter} \).

9. \( f'(x) = \frac{a}{x^2} \). Let us take the derivative: \( f''(x) = -\frac{a}{x^2} + \frac{a^2 f'(x)}{x^2} \).

Now, substitute \( f(\frac{x}{a}) = \frac{x f'(x)}{a} \) and \( f'(\frac{x}{a}) = \frac{x f''(x)}{a} \). \( f''(x) = -\frac{f'(x)}{x} + \frac{f''(x)}{x} \).

Clear denominators: \( x f(x) f''(x) + f(x) f'(x) = x f'(x) \).

Divide by \( f(x)^2 \). \( 0 = f'(x) + f''(x) f(x) = f'(x) \).

Thus, \( f'(x) = \frac{d}{x} x \) and \( f(x) = c x d \).

10. By the Mean Value Theorem there exists \( c_1 \in [-a, 0] \) such that \( |f'(c_1)| = f(0)-f(-a)| \leq \frac{2}{a} \) and consequently, \( f(c_1)^p + f'(c_1)^q \leq 1 + (\frac{2}{a})^q \). Analogously, there exists \( c_2 \in [0, a] \) such that \( f(c_2)^p + f'(c_2)^q \leq 1 + (\frac{2}{a})^q \) and \( f(x) \geq 1 - x. \) Analogously, \( 1 \geq f'(c) = \frac{f(2)-f(3)}{2-x} = \frac{1-f(x)}{2-x} \) and \( f(x) \geq |x-1| \).